Relational modal interpretation for relativistic quantum field theories

Gyula Bene*

Institute of Theoretical Physics, Eötvös University,

*Pázmány Péter sétány 1/A, H-1117 Budapest, Hungary**

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The relational version of the modal interpretation offers both a consistent quantum ontology and solution for quantum paradoxes within the framework of nonrelativistic quantum mechanics. In the present paper this approach is generalized for the case of relativistic quantum field theories. Physical systems are defined as Hilbert spaces. The concept of the reduced density matrix is also generalized so that its trace may become smaller than one, expressing the possibility of annihilation. Superselection rules are shown to follow if the whole Universe has a definite electric charge, barionic number and leptonic number.

I. INTRODUCTION

Modal interpretations aim at extracting a consistent pysical picture out of the formalism of quantum mechanics rather than relying on assumptions about an *a priori* classical world [1]. The Dieks-Vermaas version of the modal interpretations utilizes the Schmidt states, i.e., the eigenstates of the reduced density matrix, and identifies them with the actually existing, physical states. The significance of the Schmidt bases has previously been emphasized - in connection with decoherence and Everett's many worlds interpretation - by Zeh [3]. The no-go theorem of Vermaas [2] stating the impossibility of defining probabilities for the simultaneous existence of certain physical states can be understood within the framework of an interpretation which can be called the relational version of the modal interpretation [7].* Relational ideas have a long history. They appeared first in the original version of Everett's interpretation [4], then, in different forms, in [5] and in [6]. The essential idea is that states do not exist in an absolute sense but can only be defined with respect to another system (or another state [4]). This idea has been implemented in [7] in a way mathematically different from the previous propositions. The quantum reference systems here contain the system to be described. The physical states defined in the Dieks-Vermaas interpretation can be identified by the states of a system with respect to itself (i.e., when the quantum reference system coincides with the system to be described). The no-go theorem of Vermaas means now that certain states of different systems that are defined with respect to different quantum reference systems cannot be compared, not even in principle. This circumstance has been shown to be consistent with the experimental possibilities which are available according to the theory, on the other hand, it clearly goes beyond the usual ontology. Indeed, one expects that existing things, even if they are defined with respect to different reference systems, must somehow be comparable. This expectation is actually based on classical experience, and its failure does not violate any well founded physical principle. The quantum ontology emerging from the relational modal interpretation states that even the existence of the states cannot be imagined independently of the quantum reference systems. One cannot think of reality as a big book where all the states of any systems with respect to any quantum reference systems are carefully registered. Such a registration would readily imply that the simultaneous existence of any states can always be checked, i.e., any states are comparable. Precisely this is impossible. This startling statement of quantum ontology is closely related to the fact that the state of the whole Universe (this would be the "big book") does not determine uniquely the state of a subsystem with respect to itself, only a set of possible states and their probabilities. On the other hand, this fundamental change of the ontology, i.e., of the very concept of realism, is necessary in view of Bell's theorem [8]. By now it is well known that Bell's theorem and the corresponding experiments which convincingly support quantum mechanical predictions [9] imply that at least one fundamental concept should be given up or modified, either locality, causality or realism†. Modal interpretations satisfy all the requirements of locality and causality, thus the remaining option is that the concept of reality should be modified. Indeed, it has been shown that accepting the above quantum ontology Bell's inequality does not follow [10]. It is instructive to

*This has been independently developed and it turned out later that it involves both the essential ideas of the modal interpretations and those of the relational interpretations.

†Sometimes other concepts like scientific inference are also questioned.

consider the famous Einstein-Podolsky-Rosen (EPR) reality criterion [11] from our point of view. This criterion states that *If, without in any way disturbing a system, the value of a physical quantity can be predicted with unit probability, then there exits an element of the physical reality that corresponds to this quantity.* Now the point is that the value of a physical quantity depends on the state of the system, and this state must be given with respect to some quantum reference system. Thus, the EPR criterion is valid only if neither the system itself, nor the quantum reference system is disturbed. But in case of the EPR paradox [11], the quantum reference system is disturbed, so the EPR criterion is not applicable and the conclusion about the incompleteness of quantum mechanics does not follow [12]. Note that already Bohr has claimed (albeit using different arguments) that the concept of realism changes in quantum mechanics [13].

All the above considerations has been done within the framework of nonrelativistic quantum mechanics. In the present paper the relational modal interpretation [7] is generalized to the case of relativistic field theories. Such generalizations of the Dieks-Vermaas version of the modal interpretation has already been proposed in Refs. [14], [15]. In Section II. the concept of the physical systems is given and discussed. Section III. contains the main result, i.e., the generalized postulates of the interpretation. These replace von Neumann's measurement postulates, thus making quantum theory self consistent, i.e., removing the necessity of an *a priori* classical background. In Section IV. the possible origin of the superselection rules is discussed. In the concluding Section V. a summary of the results is given.

II. THE CONCEPT OF THE PHYSICAL SYSTEMS

In nonrelativistic quantum mechanics physical systems might be specified by the particles they contain. This definition becomes unsatisfactory, however, when macroscopic systems are concerned. Indeed, a description of a macroscopic system should contain its structure as well, which is not included if only its constituent particles are given. Moreover, this structure is much more important than the precise number of the particles. A straightforward possibility is to specify a system by the collection of states which correspond to the structure and functionality of that system. These states may even contain a different number of particles. Certainly, the superposition must be respected (at least to an extent allowed by the superselection rules), thus arbitrary linear combinations of these states are also allowed. This makes the collection of the states a vector space. As the scalar product of these states are defined as usually, we have an Eucledian vector space. Finally, the completion of this space gives rise to a Hilbert space, which is much narrower than the total Hilbert space of all the constituent particles. E.g., when constructing the Hilbert space of a measuring device as described above, one does not include states which correspond to a destructed device. This construction can be equally well applied in case of relativistic field theories. In that case states are given in Fock space, thus typically contain superpositions of states with different occupation numbers. In the nonrelativistic case interacting systems usually can be chosen such that they preserve their identity during the interaction, while this is in general impossible in the relativistic case. Mathematically, this means that in the nonrelativistic case the interaction moves the state of the composite system within the direct product of the subsystems' Hilbert spaces, while in the relativistic case the state may leave the direct product space during the interaction. Note that this situation can appear in the nonrelativistic case as well, e.g., if a measuring device is destroyed by a too hard interaction (say, a too low measuring range has been set), the final state of the composite system can be outside of the direct product space. In the relativistic case this situation is typical which means that a system can disappear. This means that the direct product of the subsystems' Hilbert spaces is just a subspace of the composite system's Hilbert space. Certainly, for the description of the interactions one has to choose such a Hilbert space (i.e., such a composite system) which is broad enough to accomodate the state during the whole time evolution. Such a system can be called isolated (as it does not interact with the rest of the world). Strictly speaking, there is only one such system: the whole Universe itself.

Sometimes we may assume that in the absence of interactions with other systems time evolution moves the state of the system within its original Hilbert space. Even this condition can be released, as it is reasonable in case of open systems like living beings. Indeed, a living being would die at once in the absence of interactions and thus would leave the Hilbert space which defines it on the basis of its normal functions.

III. POSTULATES

Once physical systems are defined mathematically as suitable Hilbert spaces, the next technical problem is how to give the state of a system with respect to another (broader) one. Here we follow Ref. [7] and make the necessary generalizations to get consistent rules.

As in the nonrelativistic case, we postulate that the state of a system with respect to itself is a pure state,

IV. SUPERSELECTION RULES

As a check of the consistency of the present approach I show here that superselection rules follow for any system if they are valid for the whole Universe. According to Postulate xx, it is enough to show that the state of a system with respect to the whole universe is such a density matrix that is diagonal in the electric charge, barionic and leptonic number. Let us apply Postulate yy to calculate this state. Here we can choose - without restricting the generality - the states $|\xi_{A,j}>$, $|\xi_{B,k}>$ to be charge eigenstates. Now it is clear that if the state $|\Phi>$ is a charge eigenstate, all those terms in Eq.(**??**) vanish where the states $|\xi_{A,j}>$ and $|\xi_{A,k}>$ correspond to different charges. This is because charge is an additive conserved quantity. Thus, the state (**??**) is indeed diagonal in the charge. The statement can be proven in the same way for the case of barionic and leptonic number.

V. SUMMARY AND CONCLUSION

As we have seen the postulates of the relational modal interpretation can be generalized for the case of relativistic field theories. As a mathematical description of physical systems Hilbert spaces has been constructed starting from state vectors which express the structure and functionality of the system. The postulates of Ref. [7] have been generalized accordingly. Note that the present formalism offers a useful generalization of the previous approach even within the framework of the nonrelativistic case. In the relativistic case one typically has inequalties instead of the equations of the nonrelativistic case, e.g., the trace of the states is usually smaller than unity when the quantum reference system is broader than the system to be described. The postulates are consistent and, as a result of using the trace and eigenvalue equations as basic operations, they are also independent of the representation. Finally, it has been demonstrated that the postulates accomodate the superselection rules in a consistent way.

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* Electronic address: bene@poe.elte.hu
[1] van Fraassen Dieks Vermaas Bacciagaluppi Donald
[2]
[3] H.D.Zeh
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